ECE 307 – Techniques for Engineering Decisions

FINAL REVIEW

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PROBLEM 7.27

- We use the following notation for this problem –
  car A: outcome that the car is behind door A and analogous definitions for *car B* and *car C*
- Then,

$$P\{car\ A\} = P\{car\ B\} = P\{car\ C\} = \frac{1}{3}$$

  which indicates that for the car to be behind any one of the 3 doors is equally likely
- I pick door A and the host knows where the car is; the possible outcomes are:

PROBLEM 7.27

(*i*) car is behind door *C*

$$P\left\{host\ \ picks\ door\ B\ \mid\ car\ C\right\}\ =\ 1$$

(*ii*) car is behind door *A* that I picked as my choice

$$P\left\{host\ \ picks\ door\ B\ \mid\ car\ \ A\right\}\ =$$

$$P\left\{host\ \ picks\ door\ C\ \mid\ car\ \ A\right\}\ =\ \frac{1}{2}$$

(*iii*) car is behind door *B*

$$P\left\{host\ \ picks\ door\ B\ \mid\ car\ \ B\right\}\ =\ 0$$

PROBLEM 7.27

- **Now**

$$P\left\{car\ C\ \mid\ \ host\ \ picks\ door\ B\right\}\ =$$

$$\frac{P\left\{car\ C\ and\ host\ \ picks\ door\ B\right\}}{P\left\{host\ \ picks\ door\ B\right\}}\ =$$

PROBLEM 7.27

$$\frac{\underbrace{P\{host\ \ picks\ door\, B \mid car\ C\}}_{1}\ \underbrace{P\{car\ C\}}_{(1\,/\,3)}}{\left[\begin{array}{l}\underbrace{P\{host\ \ picks\ door\, B \mid car\ A\}}_{(1\,/\,2)}\ \underbrace{P\{car\ A\}}_{(1\,/\,3)} + \\ \underbrace{P\{host\ \ picks\ door\, B \mid car\ B\}}_{0}\ \underbrace{P\{car\ B\}}_{(1\,/\,3)} + \\ \underbrace{P\{host\ \ picks\ door\, B \mid car\ C\}}_{1}\ \underbrace{P\{car\ C\}}_{(1\,/\,3)}\end{array}\right]} = \frac{2}{3}$$

- ❑ Therefore, you should switch when the host reveals the goat

PROBLEM 9 – 24

- ❑ We define $\underset{\sim}{Q}$ to be the *r.v.* representing market share with values in $[0,1]$
- ❑ We are given

$$P\{\underset{\sim}{Q} > 0.22\} = P\{\underset{\sim}{Q} < 0.08\} = 0.1$$

and

$$P\{\underset{\sim}{Q} > 0.14\} = P\{\underset{\sim}{Q} < 0.14\} = 0.5$$

PROBLEM 9.24

- Therefore

$$P\{0.08 < \underset{\sim}{Q} < 0.14\} = P\{\underset{\sim}{Q} < 0.14\} - P\{\underset{\sim}{Q} < 0.08\} = 0.4$$

$$\begin{aligned} P\{0.14 < \underset{\sim}{Q} < 0.22\} &= P\{\underset{\sim}{Q} > 0.14\} - P\{\underset{\sim}{Q} > 0.22\} \\ &= P\{\underset{\sim}{Q} < 0.22\} - P\{\underset{\sim}{Q} < 0.14\} \\ &= 0.4 \end{aligned}$$

- We pick $n = 40$, $r = 6$ and obtain beta distribution data from the tableau in the Appendix

beta

$$P_b\{\underset{\sim}{Q} \leq 0.0829 | n = 40,\ r = 6\} = 0.1$$

$$P_b\{\underset{\sim}{Q} \leq 0.2249 | n = 40,\ r = 6\} = 0.9$$

PROBLEM 9.24

- Also,

$$E\{\underset{\sim}{Q}\} = \frac{r}{n} = \frac{6}{40} = 0.15$$

- However,

$$P_\beta\{\underset{\sim}{Q} \leq 0.1441 | n = 40,\ r = 6\} = 0.5$$

and therefore,

$$P_\beta\{\underset{\sim}{Q} \leq 0.15 | n = 40,\ r = 6\} > 0.5$$

PROBLEM 9.26

- The inheritance can be invested entirely in *Mac* or in *USS* and we are given that

$$P\{\text{invested in } Mac\} = 0.8$$

and so

$$P\{\text{invested in } USS\} = 0.2$$

- Each year return on investment is normal with

$$\underset{\sim}{R}_{Mac} \sim \mathcal{N}(14\%, 4\%)$$

$$\underset{\sim}{R}_{USS} \sim \mathcal{N}(12\%, 3\%)$$

and the yearly returns are independent *r.v.s.*

PROBLEMS 9.26 (*a*)

- We compute then

$$P\left\{.06 < \underset{\sim}{R} < .18 \,\middle|\, \text{investment in } Mac\right\}$$

$$= P\left\{\frac{.06-.14}{.04} < \underset{\sim}{Z} < \frac{.18-.14}{.04}\right\}$$

$$= P\left\{-2 < \underset{\sim}{Z} < 1\right\}$$

$$= 0.8185$$

PROBLEMS 9.26 (a)

- Similarly

$$P\left\{.06 < \underset{\sim}{R} < .18 \middle| \text{investment in } USS\right\}$$

$$= P\left\{\frac{6-12}{3} < \underset{\sim}{Z} < \frac{18-12}{.3}\right\}$$

$$= P\left\{-2 < \underset{\sim}{Z} < 2\right\}$$

$$= 0.9544$$

PROBLEM 9.26 (b)

- Then, the unconditional probability is

$$P\left\{6 < \underset{\sim}{R} < 18\right\} = P\left\{6 < \underset{\sim}{R} < 18 \middle| Mac\right\} P\left\{Mac\right\} +$$

$$P\left\{6 < \underset{\sim}{R} < 18 \middle| USS\right\} P\left\{USS\right\}$$

$$= 0.8185(0.8) + 0.9544(0.2)$$

$$= 0.84568$$

PROBLEM 9.26 (c)

- We are given $P\left\{\underset{\sim}{R} > 12\right\}$ and wish to compute $P\left\{\text{investment in } Mac \middle| \underset{\sim}{R} > 12\right\}$
- We compute

$$P\left\{\underset{\sim}{R} > 12 \middle| Mac\right\} = P\left\{\underset{\sim}{Z} > \frac{12-14}{4}\right\} = P\left\{\underset{\sim}{Z} > -0.5\right\}$$

$$= 0.6915$$

and

$$P\left\{\underset{\sim}{R} > 12 \middle| USS\right\} = P\left\{\underset{\sim}{Z} > \frac{12-12}{3}\right\} = P\left\{\underset{\sim}{Z} > 0\right\}$$

$$= 0.5$$

PROBLEM 9.26 (c)

- Then $P\left\{Mac \middle| \underset{\sim}{R} > 12\right\} =$

$$\frac{P\left\{\underset{\sim}{R} > 12 \middle| Mac\right\} P\left\{Mac\right\}}{P\left\{\underset{\sim}{R} > 12 \middle| Mac\right\} P\left\{Mac\right\} + P\left\{\underset{\sim}{R} > 12 \middle| USS\right\} P\left\{USS\right\}}$$

$$= \frac{(0.6915)(0.8)}{(0.6915)(0.8) + (0.5)(0.2)}$$

$$= 0.847$$

PROBLEM 9.26 (*d*)

- We are given that

$$P\{Mac\} = P\{USS\} = 0.5$$

- Then,

$$E\{\underset{\sim}{R}\} = E\{\underset{\sim}{R}|Mac\}P\{Mac\} + E\{\underset{\sim}{R}|USS\}P\{USS\}$$

$$0.13 = 0.5\{0.14 + 0.12\}$$

and

$$var\{\underset{\sim}{R}\} = (0.5)^2 var\{\underset{\sim}{R}|Mac\} + (0.5)^2 var\{\underset{\sim}{R}|USS\}$$

$$= 0.25\{(0.04)^2 + (0.03)^2\}$$

$$0.0625 = (0.5)^2(0.5)^2 \Rightarrow \sigma_R = 0.25$$

PROBLEM 9.31 (*a*)

- We know that the length *r.v.*

$$\underset{\sim}{L} \sim \mathcal{N}(5.9, 0.0365)$$

- We compute

$$P\{\text{not fit in a } 6'' \text{ envelope}\} = P\{\underset{\sim}{L} > 5.975\}$$

$$= P\left\{\underset{\sim}{Z} > \frac{5.975 - 5.9}{0.0365}\right\}$$

$$= P\{\underset{\sim}{Z} > 2.055\}$$

$$= 0.02$$

PROBLEM 9.31 (*b*)

- We have a box with $n = 20$ and a failure occurs whenever an envelope does not fit into a box:

$$P\{\text{no fit}\} = P\{\underset{\sim}{L} > 5.975\} = 0.02$$

- From the binomial distribution for $n = 20$ with $q = 0.02$ we compute the $P\{$ 2 or more no fits $\}$
- The event of two or more no fits in a population of 20 is the event of 18 or less fits

PROBLEM 9.31 (*b*)

$$P\{\text{fit}\} = 1 - P\{\text{not fit}\} = 0.98$$

$$\underbrace{P\{\underset{\sim}{R} \leq 18\}}_{\text{number of fits out of 20}} = \underbrace{P\{\underset{\sim}{\hat{R}} \geq 2\}}_{\text{number of no fits out of 20}}$$

$$= 1 - P\{\underset{\sim}{\hat{R}} \leq 1\}$$

$$= 1 - 0.94$$

binomial (20; 0.02)

$$= 0.06$$

PROBLEM 9.31 (b)

- The interpretation of the $.06$ is as follows: we have the result that we expect, on average, that $6\ \%$ of the boxes contain 2 or more cards that do not fit the envelopes

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- On average, 7.5 people arrive in 30 minutes since

$$\frac{30\, min}{4\, min\, /\, person} = 7.5\ persons$$

and so we have the number of arriving people $\underset{\sim}{X}$ as an $r.v.$ with

$$\underset{\sim}{X} \sim Poisson\left(m = 7.5\right)$$

- A simplistic way to solve the problem is to view the individual 40% preference of each arriving

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person to be independent of the arrivals and then treat the number of arriving persons who prefer the new recipe as a *r.v.* $\underset{\sim}{P}$ with mean $(40\%)(7.5) = 3$ and so

$$\underset{\sim}{P} \sim Poisson\left(m = 3\right)$$

- Table look up produces

$$P\left\{\underset{\sim}{P} \geq 4\right\} = 0.353$$

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- A more rigorous approach is to treat the performance of each arrival as a binomial

$$\underset{\sim}{X} = number\ of\ arrivals\ in\ 30\ minutes \sim Poisson\left(m = 7.5\right)$$

- Each arrival *i* has a preference $\underset{\sim}{P}_i$ for new recipe with

$$\underset{\sim}{P}_i \sim binomial\left(n = \underset{\sim}{X},\ p = 0.4\right)$$

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- We need to compute $P\left\{\sum_i \underset{\sim}{P}_i \geq 4\right\}$
- We condition over the number of arrivals

$$P\left\{\sum_i \underset{\sim}{P}_i \geq 4\right\} = \sum_{n=1}^{\infty} P\left\{\sum_{i=1}^{n} \underset{\sim}{P}_i \geq 4 \,\middle|\, \underset{\sim}{X} \geq n\right\} P\left\{\underset{\sim}{X} = n\right\}$$

$$= P\left\{\sum_{i=1}^{4} \underset{\sim}{P}_i \geq 4 \,\middle|\, \underset{\sim}{X} \geq 4\right\} P\left\{\underset{\sim}{X} = 4\right\} + P\left\{\sum_{i=1}^{5} \underset{\sim}{P}_i \geq 4 \,\middle|\, \underset{\sim}{X} \geq 5\right\} P\left\{\underset{\sim}{X} = 5\right\} + P\left\{\sum_{i=1}^{6} \underset{\sim}{P}_i \geq 4 \,\middle|\, \underset{\sim}{X} \geq 6\right\} P\left\{\underset{\sim}{X} = 6\right\} + \ldots$$

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- Note that $P\left\{\sum_{i=1}^{n} \underset{\sim}{P}_i \geq 4 \,\middle|\, \underset{\sim}{x} \geq 4\right\} P\left\{\underset{\sim}{X} = n\right\}$ is simply the binomial distribution value with parameters $(n, 0.4)$ and $P\left\{\underset{\sim}{X} = n\right\}$ is the Poisson distribution value with $m = 7.5$
- The sum has insignificant contributions for $n > 16$

10.12: PROBLEM FORMULATION

- This is a multi-period planning problem with a 7-month horizon
- Define the following for use in backward regression
  - stage: a month in the planning period
  - state variable: the number of crankcases S_n left over from the stage $(n-1)$, $n = 1, 2, \ldots, N$ with $S_7 = 0$ $\left(\textit{initial stage}\right)$ and S_0 *unspecified*

10.12: PROBLEM FORMULATION

- decision variables: purchase amount d_n for stage n, $n = 1, 2, \ldots, 7$
- transition function: the relationship between the amount in inventory, purchase decision and demand in stages n and $(n-1)$

$$S_{n\text{-}1} = S_n + d_n - D_n \qquad n = 1, 2, \ldots, N$$

where,

$$D_n = \text{demand at stage } n \qquad n = 1, 2, \ldots, N$$

10.12: PROBLEM FORMULATION

- return function: costs of purchase in stage n plus the inventory holding costs, with the mathematical expression

$$f^*_n(S_n) = C_n + (S_n + d_n - D_n)\,0.50 + f^*_{n\text{-}1}(S_{n\text{-}1})$$

costs of lot size ordered *per unit inventory charges*

and

$$f^*_0(S_0) = 0$$

10.12: STAGE 1 SOLUTION

$$D_1 = 600$$

$$f^*_1(S_1) = \min_{d_1}\{C_1 + (S_1 + d_1 - D_1)\,0.50\}$$

S_1	value of f_1 for d_1				$f^*_1(S_1)$	d^*_1
	0	500	1000	1500		
0			5200	7950	5200	1000
100		3000	5250	8000	3000	500
200		3050	5300	8050	3050	500
300		3100	5350	8100	3100	500
400		3150	5400	8150	3150	500
500		3200	5450	8200	3200	500
600	*0*	3250	5500	8250	*0*	*0*

10.12: STAGE 2 SOLUTION

$D_2 = 1200$

$$f_2^*(S_2) = \min_{d_2}\{C_2 + (S_2 + d_2 - D_2)0.50 + f_1^*(S_2 + d_2 - D_2)\}$$

S_2	value of f_2 for d_2				$f_2^*(S_2)$	d_2^*
	0	500	1000	1500		
0				10750	10750	1500
100				10850	10850	1500
200			10200	10950	10200	1000
300			8050	7800	7800	1500
400			8150		8150	1000
500			8250		8250	1000
600			8350		8350	1000

10.12: STAGE 3 SOLUTION

$D_3 = 900$

$$f_3^*(S_3) = \min_{d_3}\{C_3 + (S_3 + d_3 - D_3)0.50 + f_2^*(S_3 + d_3 - D_3)\}$$

S_3	value of f_3 for d_3				$f_3^*(S_3)$	d_3^*
	0	500	1000	1500		
0			15900	16150	15900	1000
100			15300		15300	1000
200			12950		12950	1000
300			12350		13350	1000
400		11050	13500		11050	500
500		13900	13650		13650	1000
600		13300			13300	500

10.12: STAGE 4 SOLUTION

$D_4 = 400$

$$f_4^*(S_4) = \min_{d_4}\left\{C_4 + (S_4 + d_4 - D_4)0.50 + f_3^*(S_4 + d_4 - D_4)\right\}$$

S_4	value of f_4 for d_4: 0	500	1000	1500	$f_4^*(s_4)$	d_4^*
0		18350	18600		18350	500
100		16050			16050	500
200		16500			16500	500
300		14250			14250	500
400	15900	16900			15900	0
500	15350	16600			15350	0
600	13050				13050	0

10.12: STAGE 5 SOLUTION

$D_5 = 800$

$$f_5^*(S_5) = \min_{d_5}\left\{C_5 + (S_5 + d_5 - D_5)0.50 + f_4^*(S_5 + d_5 - D_5)\right\}$$

S_5	value of f_5 for d_5: 0	500	1000	1500	$f_5^*(S_5)$	d_5^*
0			21600		21600	1000
100			19400		19400	1000
200			21100		21100	1000
300		21350	20600		20600	1000
400		19100	18350		18350	1000
500		19600			19600	500
600		17400			17400	500

10.12: STAGE 6 SOLUTION

$D_6 = 1100$

$$f_6^*(S_6) = \min_{d_6}\{C_6 + (S_6 + d_6 - D_6)0.50 + f_5^*(S_6 + d_6 - D_6)\}$$

S_6	value of f_6 for d_6				$f_6^*(S_6)$	d_6^*
	0	500	1000	1500		
0				26050	26050	1500
100			26650	27350	26650	1000
200			24500	25200	24500	1000
300			26250		26250	1000
400			25800		25800	1000
500			21300		21300	1000
600		24600	20650		20650	1000

10.12: STAGE 7 SOLUTION

- For stage 7, $D_7 = 700$ and

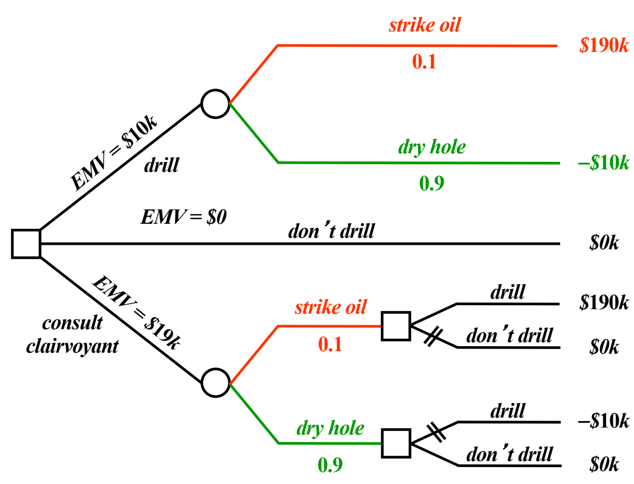
$$f_7^*(S_7) = \min_{d_7}\{C_7 + (S_7 + d_7 - D_7)0.50 + f_6^*(S_7 + d_7 - D_7)\}$$

- Optimal total cost over 7 months = $ 31,400

  obtained using the purchasing policy below

month	1	2	3	4	5	6	7
amount of material	1000	1000	1000	0	1000	1500	500

12.7: OIL WILDCATTING PROBLEM: DECISION TREE



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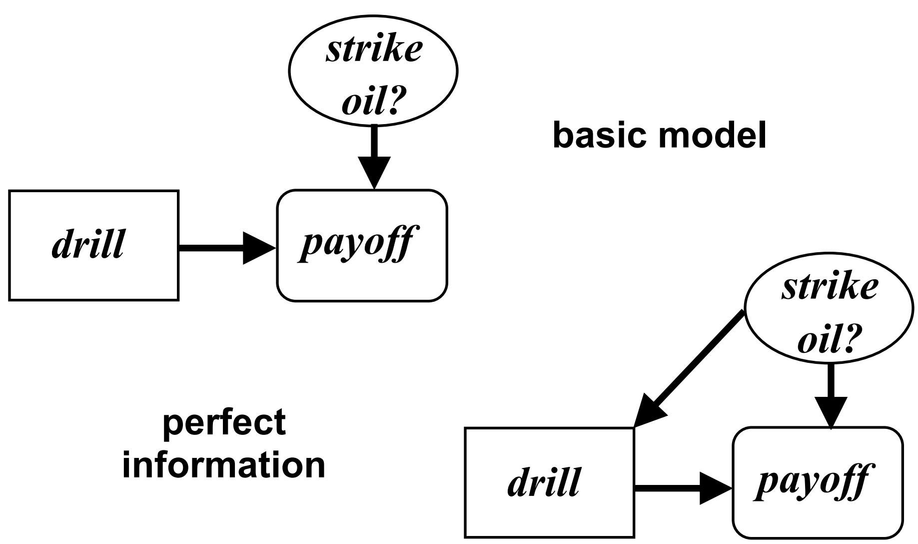
12.7: BLOCK DIAGRAMS



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12.7 *EVPI* AND *EVII*

- We evaluate the expected value of the clairvoyant information

$$EVPI = \underbrace{EMV\,(\,clairvoyant\,)}_{\$19k} - \underbrace{EMV\,(\,drill\,)}_{\$10k} = \$9k$$

- We have the following conditional probabilities

$P\,\{\text{“good”}\,|\,oil\,\} = 0.95$ and $P\,\{\text{“poor”}\,|\,dry\,\} = 0.85$

- We are also given that

$P\,\{dry\} = 0.9$ and $P\,\{\,oil\,\} = 0.1$

- We can find $P\,\{\text{“good”}\}$ and $P\,\{\text{“poor”}\}$ with the

12.7 *EVPI* AND *EVII*

law of total probability

$$P\,\{\text{“good”}\,\} = P\,\{\text{“good”}\,|\,oil\,\}\,P\,\{oil\} \quad +$$

$$P\,\{\text{“good”}\,|\,dry\}\,P\,\{dry\} \quad =$$

$$(0.95)\,(0.1) + (0.15)\,(0.9) = 0.23$$

$$P\,\{\text{“poor”}\} = 1 - P\,\{\text{“good”}\} = 1 - 0.23 \quad = \quad 0.77$$

12.7 *EVPI* AND *EVII*

- Now we can find

$$P\{oil \mid "good"\} = \frac{P\{"good" \mid oil\}\, P\{oil\}}{\begin{bmatrix} P\{"good" \mid oil\}\, P\{oil\} + \\ P\{"good" \mid dry\}\, P\{dry\} \end{bmatrix}}$$

$$= \frac{(0.95)(0.1)}{(0.95)(0.1)+(0.15)(0.9)}$$

$$= 0.41$$

$$P\{dry \mid "good"\} = 1 - P\{oil \mid "good"\} = 0.59$$

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12.7 *EVPI* AND *EVII*

and

$$P\{oil \mid "poor"\} = \frac{P\{"poor" \mid oil\}\, P\{oil\}}{\begin{bmatrix} P\{"poor" \mid oil\}\, P\{oil\} + \\ P\{"poor" \mid dry\}\, P\{oil\} \end{bmatrix}}$$

$$= \frac{(0.05)(0.1)}{(0.05)(0.1)+(0.85)(0.9)}$$

$$= 0.0065$$

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